

**The Contraceptive Revolution and the Second Demographic Transition:  
An Economic Model of Sex, Fertility, and Marriage**

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## **The Contraceptive Revolution and the Second Demographic Transition: An Economic Model of Sex, Fertility, and Marriage<sup>1</sup>**

*Although several papers have documented the labor-market effects of the Pill (and increased contraceptive efficacy), none has provided a sound micro-foundation for these results. In this paper we develop a two-stage model with endogenous household decisions regarding sex, fertility, marriage, and the consumption of other goods. In the second-stage model, households choose sex, children, and consumption of other goods to maximize utility subject to a budget and a fertility constraint. This leads to an indirect utility function that depends on prices, income, and contraceptive efficacy. In the first-stage problem, households choose their marital status based on the expected utility of being married or unmarried from the second-stage problem. The marriage decision is based on the difference in the expected utility of married and unmarried states. Changes in behavior in response to marginal changes in contraceptive efficacy are separated into substitution effects, sex effects, and child effects. We find that an increase in contraceptive efficacy, such as availability of the Pill, generally leads to increased sexual activity but has ambiguous effects on the children per household, where married households will have fewer children and unmarried households will have more. Such changes generally lead to a greater increase in sexual activity among unmarried households than among married ones. Also, increases in contraceptive efficacy tend to diminish the difference in utility between married and unmarried households, leading to lower marriage rates and higher divorce rates. These results correspond to the recent historical trends which characterize the so-called “second demographic transition”, including declining overall fertility rates, increasing non-marital fertility, and the decline in marriage.*

Keywords: microeconomic theory, contraception, sex, marriage, divorce, cohabitation, fertility

JEL Classification: D01, J12, J13, Z13

### **Introduction**

The first demographic transition, beginning around the early 1800s, was characterized especially by substantial declines in both mortality and fertility. This transition generally benefitted children through better health and physical welfare, greater familial stability, and increased access to education. But the second demographic transition, beginning at or around 1960, has so far proved less beneficial to children, as argued by a growing numbers of

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<sup>1</sup> The term “second demographic transition” was originally coined by Lesthaeghe and Van de Kaa in a book chapter entitled “Twee demografische transitie?” (Two demographic transitions?).

sociologists and demographers.<sup>2</sup> The key features of the second demographic transition include rising marital instability (through cohabitation and divorce), enormous upward swings in non-marital childbearing, and delays in marriage and marital childbearing. Among these, the trend in non-marital childbearing remains the most puzzling in light of the simultaneous advances in reproductive technologies, especially the Pill and legal abortion, widely available to unmarried women since the late 1960s.

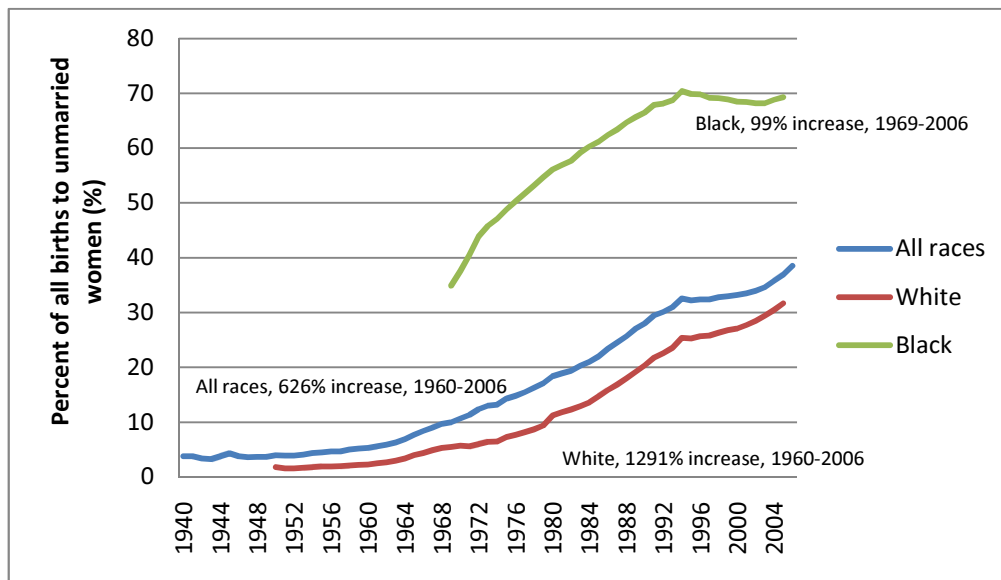


Figure 1. Percent of births to unmarried women, 1940-2006<sup>3</sup>

Figure 1 illustrates that from 1960 to 2006 the percent of all births to unmarried women in the United States increased 626%.<sup>4</sup> Although the total percentage of unmarried births in the black community is twice that among white women, the actual percent increases have been largest among the non-black populations, indicating that the growth in non-marital fertility has not been driven by the minority population. Rather, whatever is driving the upward trend in non-marital births appears to affect the United States broadly and with respect to every racial and

<sup>2</sup> McLanahan, 2004

<sup>3</sup> Brady, Martin, & Ventura, 2007; also, Ventura & Bachrach, 2001

<sup>4</sup> Ibid.

ethnic subgroup. What can explain a shift in childbearing patterns on this order of magnitude? And why should we care?

To address the second question, we should care because in spite of improvements to various social welfare programs, assistance to fragile families, and other non-governmental charitable assistance, children born to unmarried parents are more likely to be born into poverty, less likely to ever live with their two biological parents, and more likely to engage in various risky behaviors at an early age. They are also emotionally and educationally disadvantaged.<sup>5</sup> This has been a concern for policy makers since as early as 1935 with the initiation of Aid to Dependent Children (ADC). The program was conceived to help children suffering from an unemployed, deceased or incapacitated parent, and to prevent their placement in orphanages, though eventually it was expanded to include aid to children born to unmarried mothers. Since that time, a multitude of governmental programs have tried to bridge the gap for children born to single mothers, including the welfare reforms of 1996. But the disadvantages for children born to unmarried parents remain.

At the same time, there have been ongoing programs to reduce teen pregnancy for several decades at both state and local levels, most prominently through schools, but single childbearing is not primarily a “teenage” phenomenon anymore, even though in 1970 half of non-marital births were to women under the age of 20. Today only 23% of non-marital births are to teenagers, and the biggest increases in non-marital fertility since 1970 have been in adult populations.<sup>6</sup> Unfortunately, children born to older unmarried women do not fare better than

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<sup>5</sup> Amato and Keith, 1991; McLanahan and Sandefur, 1994; Edin and Kefalas, 2005

<sup>6</sup> Ventura, 2009

those born to unmarried teenage women and the socioeconomic consequences are not substantively different.<sup>7</sup>

Policy has been largely ineffective at reversing the broad trend in non-marital births. According to the US Census Bureau, approximately 17 million children are living with mothers only, nearly 25% of all children.<sup>8</sup> Given the weighty cost for these children, and the apparent unresponsiveness of the trend to various policy measures, it seems critical to explore the reasons for this particularly puzzling aspect of the second demographic transition.

The basic logical difficulty in understanding the cause of the increase in non-marital births is that it occurs contemporaneously with another trend which should have had the opposite result: the diffusion of access to reliable birth control, especially the Pill, for unmarried women.<sup>9</sup> In the literature there have been three prevailing ways of reconciling these apparently conflicting trends. The first has been to stipulate some common cause of the two trends, such as a cultural change, or economic factors such as women's wages or the structure of welfare payments. Classic papers include Thornton, Axinn and Hill (1992), and also Lesthaeghe (1983), Lesthaeghe and Surkyn (1988), and Lesthaeghe and Wilson (1985). Also important is Bumpass (1990).

The second has been to examine the changes in birth control access as a technology shock, and consider how the shock might have had differential effects on different populations, generating in some instances the expected result (fewer births) and in other instances the counter-intuitive result (more births). These explanations tend to involve differential technology

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<sup>7</sup> Geronimus and Korenman, 1992, 1993

<sup>8</sup> 2004, US Census Bureau, SIPP, Wave 2.

<sup>9</sup> While the FDA approved the first birth control pill (Enovid) in 1960, various state-level statutes banned the sale of the Pill even to married women until 1965 the U.S. Supreme Court overturned a Connecticut law prohibiting the sale of contraceptives. In 1972 the U.S. Supreme Court overturned a Massachusetts law prohibiting the sale of contraceptives to unmarried women. It is generally maintained that a majority of U.S. women had access to the Pill by the early 70s. See Goldin and Katz, 2002, and Bailey, 2009, for thorough treatment of the diffusion and adoption of the Pill among unmarried and married women respectively.

adoption, or differential use, yielding higher rates of unwanted births (or accidental fertility) among different populations. The perennial paper is Akerlof, Yellen and Katz (1996) but sociologists such as Paula England have produced distinct but complementary work looking at differential unwanted fertility rates by educational status.<sup>10</sup>

The third approach has been to take account of mounting qualitative data suggesting that many non-marital births are not “unwanted” births, but may be better understood as “re-timed” births. In this view, the technology shock initially alters constraints making new allocations of marriage and childbearing ideal given new constraints. Sociologists Kathryn Edin and Maria Kefalas have explored this possibility qualitatively, arguing that observed patterns of non-marital childbearing for low-income women should not necessarily be viewed as unwanted or ill-timed, but rather optimally timed given various life constraints.<sup>11</sup> She writes, “Since these mothers generally believe that childbearing and rearing young children necessitate a temporary withdrawal from the labor market, many place the ideal age at which to marry in the late 20s (when their youngest child is school age) and the ideal age to bear children in the early 20s—the age they say is the “normal” time for women to have children. Delaying marriage until they can concentrate more fully on labor market activity maximizes their chances of having a marriage where they can have equal bargaining power.”<sup>12</sup> Work by Geronimus (1997) and Geronimus and Korenman (1992) also supports a re-timing hypothesis. Such a view re-orientes the rising trend in non-marital fertility from a perverse outcome whereby some groups benefit more than others from a technological advancement, to a socially problematic outcome (from the perspective of

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<sup>10</sup> See for example Musick, England, Edgington, and Kangas, 2007

<sup>11</sup> See especially Edin and Kefalas, 2005; also Edin 2000

<sup>12</sup> Edin, 2000

children) whereby non-marital fertility is explained as a rational response to changing incentives. Optimal policy will depend on which view is a better characterization of this puzzling trend.<sup>13</sup>

In this paper, we explore whether the latter view—that contraceptive technology allowed a re-timing of births among low-income women—might be sustained within the familiar neo-classical framework. In the canonical economic treatment of the family, Becker (1981) argues that the contraceptive revolution cannot account for the rapid changes in marital and non-marital fertility now recognized as part of the second demographic transition. We counter this by introducing a fertility constraint that relates sexual activity and births into a model of household decisions regarding marriage, sex, children, and consumption of other goods. Contraceptive technology affects the model by weakening the fertility constraint, so that households with more effective contraception can have more sexual intercourse for a given number of children, or fewer children for a given amount of sexual activity. We model advancements in contraceptive technology, such as the Pill, or access to abortion, as exogenous changes in the efficacy of contraception.

## **Model**

The model is separable into two stages: the second-stage model is a model for demand for sex, children, and other goods, and the first-stage model is a model of the choice to marry or not. The decision to marry is determined by the difference in utilities between married and unmarried households. The difference in utilities is ordinarily generated by the income effect, as married

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<sup>13</sup> For example, if the former view is correct, and non-marital births are generally the result of non-adoption of reproductive technology (or poor usage thereof), a good strategy for the reduction of non-marital births would be to focus on the non-adopters (or poor users) through educational initiatives and incentives for use. On the other hand, if the non-marital births are optimally timed from the perspective of low-income women, funding for increased distribution of contraceptives and educational programs would be ill-spent. Instead, policy would be better aimed at remedying the underlying constraints which make non-marital fertility the optimal choice for so many.

households have more resources, or income, than unmarried households, which leads to higher utility and higher demand for sex, children, and other goods, all of which are assumed to be normal.

### *Sex and Children*

Let  $s$ ,  $k$ , and  $x$  be demand for sex, children (kids), and other goods, respectively, with corresponding prices  $p_s$ ,  $p_k$ , and  $p_x$ . Let  $m$  be income,  $\beta$  be the natural productivity of sex, and  $z$  be the efficacy of contraception,  $z \in [0,1]$ , with  $z = 0$  being no contraception and 1 being sterilization. An increase in  $z$  means an increase in the efficacy of contraception. Consider the household's problem

$$\begin{aligned} v(p, m, z) &= \max_{s,k,x} u(s, k, x) \\ \text{s. t. } &p_s s + p_k k + p_x x \leq m \\ &(1 - z)\beta s \leq k. \end{aligned}$$

Households choose sex, children, and consumption of other goods so as to maximize utility subject to the budget constraint and a fertility constraint relating sex to the number of children they desire to have. Non-negativity constraints on sex, children, and other goods are implicit. The indirect utility function is a function of prices, income, and contraceptive efficacy, which is exogenous to the model. We assume that a solution exists for all choices of  $z$ . With complete sterilization,  $z = 1$  and the problem reduces to the ordinary consumer's problem.

What is meant by contraceptive efficacy,  $z$ ? This parameter includes not just the technological efficacy, but also knowledge of how to use contraceptive technology, the availability of such technology to households, and the willingness to use it.<sup>14</sup> The efficacy of

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<sup>14</sup> Knowledge and education are an important part of this parameter. We will argue that changes in contraceptive efficacy changed fertility in the 1800s through these means primarily, and not (as it did later) through huge shocks in technology or availability.

contraception increases as technology improves, as it becomes more widely available, and as households are more willing and able to use it. A corollary will be that the contraceptive efficacy parameter also increases with the multiplication of family planning clinics and with the proliferation of sex education classes when such education includes components to teach students how to use contraception, informs them where it is available, and encourages them to use it.<sup>15</sup>

Let  $\lambda$  and  $\mu$  be the Lagrange multipliers on the budget and the fertility constraint, so the Lagrangian is

$$L = u(s, k, x) + \lambda(m - p_s s - p_k k - p_x x) + \mu(k - (1 - z)\beta s).$$

In addition to the constraints, the first-order conditions for this problem are

$$\begin{aligned} u_s - \lambda p_s - \mu(1 - z)\beta &= 0 \\ u_x - \lambda p_x &= 0 \\ u_k - \lambda p_k + \mu &= 0. \end{aligned}$$

The cost of sex includes the direct cost  $p_s$  plus the indirect costs of having children, and the cost of children is the direct cost  $p_k$  minus the indirect benefits of sex. However, the direct costs of sex,  $p_s$ , are likely to be trivially small, so decisions about sex will tend to be dominated not by the direct costs of sex but the indirect cost of children. Notice that the fertility constraint effectively increases the cost of sex above  $p_s$ , and decreases the cost of children below  $p_k$ . This leads to the following ordering

$$\frac{u_s}{p_s} > \frac{u_x}{p_x} > \frac{u_k}{p_k}.$$

When  $z < 1$  and the fertility constraint is binding, this ordering of marginal utilities per dollar must hold for all feasible prices in order for a maximum to exist. In particular, this order must

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<sup>15</sup> There is a strong correlation between education and willingness to use contraception—this is robust across countries.

hold in the state of nature, i.e. when no contraception is used, or  $z = 0$ . If this ordering is a property of utility functions in general, then the implications of sterilization for the model are readily apparent: sterilization removes the fertility constraint, so utility maximizing households will have as much sex as possible, spend the remainder of the budget on consumption of other goods, and have no children. This is true of both sterilization and of any contraceptive practice that is as efficacious as sterilization.

How does demand for sex change as contraception becomes more efficacious? This change can represent either the introduction of a new contraceptive technology, such as the Pill, or the broad historical trend of increasing contraceptive availability, education, and efficacy observed especially over the past two centuries. The system of equations for comparative statics analysis is

$$\begin{bmatrix} u_{ss} & u_{sk} & u_{sx} & -p_s & -(1-z)\beta \\ u_{ks} & u_{kk} & u_{kx} & -p_k & 1 \\ u_{xs} & u_{xk} & u_{xx} & -p_x & 0 \\ -p_s & -p_k & -p_x & 0 & 0 \\ -(1-z)\beta & 1 & 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} \partial s^*/\partial z \\ \partial k^*/\partial z \\ \partial x^*/\partial z \\ \partial \lambda^*/\partial z \\ \partial \mu^*/\partial z \end{bmatrix} = \begin{bmatrix} -\mu\beta \\ 0 \\ 0 \\ 0 \\ -\beta s \end{bmatrix}$$

Let  $|H|$  be the determinant of the left-hand side matrix.  $H$  is a third-order bordered principal minor, so the second-order sufficiency conditions imply that  $|H| < 0$ . The comparative statics for the change in demand are

$$\begin{aligned} \frac{\partial s^*}{\partial z} &= \frac{1}{|H|} \left[ -\mu\beta p_x^2 - (1-z)\beta^2 s^* \begin{vmatrix} u_{kk} & u_{kx} & -p_k \\ u_{xk} & u_{xx} & -p_x \\ -p_k & -p_x & 0 \end{vmatrix} - \beta s^* \begin{vmatrix} u_{sk} & u_{sx} & -p_s \\ u_{xk} & u_{xx} & -p_x \\ -p_k & -p_x & 0 \end{vmatrix} \right] \\ &= \frac{1}{|H|} \left[ -\mu\beta p_x^2 - \beta k^* \begin{vmatrix} u_{kk} & u_{kx} & -p_k \\ u_{xk} & u_{xx} & -p_x \\ -p_k & -p_x & 0 \end{vmatrix} - \beta s^* \begin{vmatrix} u_{sk} & u_{sx} & -p_s \\ u_{xk} & u_{xx} & -p_x \\ -p_k & -p_x & 0 \end{vmatrix} \right] \\ &= \frac{-\mu\beta}{\lambda} \frac{\partial s^H}{\partial p_s} + \beta k^* \frac{|\hat{H}|}{|H|} \frac{\partial \hat{s}}{\partial s_h} - \beta s^* \frac{|\tilde{H}|}{|H|} \frac{\partial \tilde{s}}{\partial k_l}. \end{aligned}$$

The determinants  $|\hat{H}|$  and  $|\tilde{H}|$  are for analytical matrices for related maximization problems defined in the appendix; both are negative. There are three terms in this derivative: (i) a

substitution effect, (ii) a sex effect, and (iii) a child effect. As the efficacy of contraception increases, sex is cheaper, households can have more sex for a given number of children, and they can have fewer children for a given amount of sex: cheap sex, more sex, fewer kids. The first two effects are positive and the third is ambiguous.

The substitution effect is analogous to the ordinary substitution effect in consumer theory: an increase in  $z$  relaxes the fertility constraint and decreases the indirect costs of sex, leading to a substitution towards sex away from other goods. The two other effects have to do with the fact that the fertility constraint acts in two ways: it can be thought of as an upper bound to the amount of sex or as a lower bound to the number of children. The “more sex effect” is the effect of increasing the upper bound to the amount of sex, and the “fewer children effect” is the effect of reducing the lower bound to the number of children. The “more sex effect” is unambiguously positive; increasing the upper bound to sex unambiguously leads to more sex. The term in the brackets is a second-order bordered principal minor to the ordinary consumer problem and has a positive determinant by the second-order sufficiency conditions. The third term, the “fewer kids effect,” is ambiguous. Reducing the lower bound to the number of children unambiguously leads to fewer children in itself, but whether this leads to more or less sex depends on how children and sex are related in the utility function, i.e. whether they are complements, substitutes, or unrelated. These effects are likely to be trivial, so the first two effects will tend to dominate: more effective contraception leads to more sexual intercourse. In the plausible special case when the marginal utility of sex is unrelated to children and other goods and the cost of sex is zero ( $u_{sk} = u_{sx} = p_s = 0$ ), the third term is zero and the result is unambiguous.

Do marginal changes in contraceptive efficacy lead to a relatively greater impact on demand for sex in unmarried households or married ones? The first-order conditions can be rearranged to show

$$\lambda = \frac{u_x}{p_x}$$

$$\mu = \frac{u_x}{p_x} p_k - u_k$$

Substituting these into the first-order condition for sex,

$$u_s - \lambda p_s + (1 - z)\beta[u_k - \lambda p_k] = 0$$

From the earlier analysis,  $u_s - \lambda p_s > 0$  and  $u_k - \lambda p_k < 0$ : sex is a net benefit and children are a net cost. Households for whom children are a net benefit do not use contraception. From this equation, the first-order effect of an increase in contraceptive efficacy is to reduce the net cost of children. Changes in contraceptive efficacy will have a greater effect on households for which the net cost of children is higher, and a lesser effect on those for which the net cost of children is lower. Generally, the net costs of children will be higher for unmarried households and lower for married ones, so marginal improvements in contraception should lead to a larger change in demand for sex, among unmarried households than it will among married ones. If consumption of other goods exhibits diminishing marginal utility, for example, then the marginal utility of consumption of other goods,  $\lambda$ , will be lower for married households because of their higher income. Married households would then have lower direct costs of sex and indirect costs of children, so they will tend to have more sex and children than unmarried households.

We now turn to how demand for children is affected by an increase in the efficacy of contraception. Like demand for sex, the changes in demand for children in response to increasing the efficacy of contraception can be separated into a substitution effect, a sex effect, and a child effect.

$$\begin{aligned}
\frac{\partial k^*}{\partial z} &= \frac{1}{|H|} \left[ -(1-z)\mu\beta p_x^2 + (1-z)\beta^2 s^* \begin{vmatrix} u_{ks} & u_{kx} & -p_k \\ u_{xs} & u_{xx} & -p_x \\ -p_s & -p_x & 0 \end{vmatrix} + \beta s^* \begin{vmatrix} u_{ss} & u_{sx} & -p_s \\ u_{xs} & u_{xx} & -p_x \\ -p_s & -p_x & 0 \end{vmatrix} \right] \\
&= \frac{1}{|H|} \left[ -(1-z)\mu\beta p_x^2 + \beta k^* \begin{vmatrix} u_{ks} & u_{kx} & -p_k \\ u_{xs} & u_{xx} & -p_x \\ -p_s & -p_x & 0 \end{vmatrix} + \beta s^* \begin{vmatrix} u_{ss} & u_{sx} & -p_s \\ u_{xs} & u_{xx} & -p_x \\ -p_s & -p_x & 0 \end{vmatrix} \right] \\
&= \mu \frac{\partial k^H}{\partial p_s} + \beta k^* \frac{|\hat{H}|}{|H|} \frac{\partial \hat{k}}{\partial s_h} - \beta s^* \frac{|\tilde{H}|}{|H|} \frac{\partial \tilde{k}}{\partial k_l}.
\end{aligned}$$

The first term is the substitution effect, the second is the sex effect, and the third is the child effect. In this case, the substitution effect is positive, the sex effect is ambiguous, and the child effect is negative. For the substitution effect, increasing contraceptive efficacy leads to more sex and, by itself, this leads to more children, since sex and children are jointly determined by the fertility constraint. The sex effect is ambiguous, and it depends on how sex, children, and other goods are related in the utility function. In the plausible special case where sex is unrelated to children and other goods and the cost of sex is zero, i.e.  $u_{ks} = u_{xs} = p_s = 0$ , then the sex effect is zero. The child effect is the effect of relaxing the lower bound for children, which by itself leads to fewer children.

The fertility constraint can clarify the ambiguity in the change in demand for children in response to an increase in contraceptive efficacy. Assuming that the fertility constraint is binding before and after any changes, optimal demand for children is given by

$$k^* = (1-z)\beta s^*.$$

Taking the derivative with respect to the efficacy of contraception,  $z$ , shows

$$\frac{\partial k^*}{\partial z} = -\beta s^* + (1-z)\beta \frac{\partial s^*}{\partial z}.$$

The first term is negative, and the second term is positive; the result is theoretically ambiguous, but depends on the amount of sex prior to the change,  $s^*$ . In particular,

$$\frac{\partial k^*}{\partial z} < 0 \text{ when } s^* > (1 - z) \frac{\partial s^*}{\partial z} \text{ and}$$

$$\frac{\partial k^*}{\partial z} > 0 \text{ when } s^* < (1 - z) \frac{\partial s^*}{\partial z}.$$

The effect of a change in contraceptive efficacy depends on sexual activity of the household prior to the change: fewer children will be born to households that were more sexually active before the change, and more children will be born to households that were less sexually active before the change. Clearly,  $\frac{\partial k^*}{\partial z} > 0$  for  $s^* = 0$ , i.e. increasing the efficacy of contraception will lead to more children being born to households that would have otherwise been abstinent. This analysis is valid when demand for sex depends on contraceptive efficacy and the fertility constraint is binding, but it is otherwise independent of the household's utility maximization problem, so the result is very general.

Generally, since demand for sex increases with income and decreases with the cost of children, married households would have had more sex than unmarried households prior to the change in the efficacy of contraception. The two results, then, may be assumed to correspond to the married and unmarried households: for married households,  $s^*$  is relatively high and  $\frac{\partial k^*}{\partial z} < 0$ ; for unmarried households,  $s^*$  is relatively low and  $\frac{\partial k^*}{\partial z} > 0$ . So the model predicts that technology shocks which increase the efficacy of contraception should lead to fewer children born to married households and more children born to unmarried households. Where married households are a much larger fraction of the population, the net effect of these two changes would be a decline in the overall fertility rate. This is consistent with recent historical trends in both fertility and out-of-wedlock births.

## *Marriage*

We now extend the model to include the decision to marry. Why do households, or couples, choose to marry? The argument here is that households marry because of the income effect. Income includes the value of time spent either in the labor market or in the production of household goods, and the demands of children are such that the additional income or resources of another parent are desired to assist a household in satisfying those demands. Unmarried households are more likely to have fewer children or to not have children because they cannot afford them, and married households will tend to have more children because they can afford them.<sup>16</sup>

Households face the same prices— $p_s$ ,  $p_k$ , and  $p_x$ —and the same contraceptive efficacy,  $z$ . Let  $m_1$  be the income of unmarried households and  $m_2$  the income of married households,  $m_2 > m_1$ . Since the indirect utility function is increasing in income, the utility of married households will be higher than that of single households, and, in the deterministic model, all households would marry. Let  $\varepsilon_1$  and  $\varepsilon_2$  be random variables that describe the unpredictable nature of unmarried and married life, respectively, and let  $\varepsilon = \varepsilon_2 - \varepsilon_1$ . The values of the random variables are known prior to the marriage decision. Assume that  $E(\varepsilon_1) = E(\varepsilon_2) = 0$ , so  $E(\varepsilon) = E(\varepsilon_2 - \varepsilon_1) = 0$ . Let  $w$  be an indicator variable of marital status where  $w = 1$  indicates marriage, so households marry according to

$$\max_w (1 - w)[v(p, m_1, z) + \varepsilon_1] + w[v(p, m_2, z) + \varepsilon_2]$$

Households marry when

$$v(p, m_1, z) + \varepsilon_1 < v(p, m_2, z) + \varepsilon_2$$

or

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<sup>16</sup> There are some critical exceptions to this, discussed below.

$$v(p, m_1, z) - v(p, m_2, z) < \varepsilon$$

Households will not marry when

$$v(p, m_1, z) - v(p, m_2, z) > \varepsilon$$

Let  $y(z)$  be the difference in utility between single and married households for a given level of contraceptive efficacy,  $y(z) = v(p, m_1, z) - v(p, m_2, z)$ , and let  $F(s)$  be the cumulative distribution function of  $\varepsilon$ , where  $F'(s) > 0$ . Because married households have higher income, they have higher utility on average,  $y(z) < 0$  and  $-y(z)$  is the average utility premium that married households receive over unmarried ones. The fraction of unmarried households is the integral of  $\varepsilon$  from negative infinity to the point where  $s = y(z)$  and the integral from that point to positive infinity is the fraction of married households, i.e.  $pr(\text{unmarried}) = pr(w = 0) = F(y(z))$  and  $pr(w = 1) = 1 - pr(w = 0)$ . This static model can be thought of as a snapshot of society over a particular time period: the first inequality describes households that marry or remain married; the second households that remain unmarried or divorce.

The decision to marry or not depends on the difference between the utility of married and unmarried households. An increase in the efficacy of contraception will lead to fewer married households and more unmarried households—single or divorced—when

$$F' \left[ \frac{dv(p, m_1, z)}{dz} - \frac{dv(p, m_2, z)}{dz} \right] = F' \frac{dy}{dz} > 0.$$

$F'(s) > 0$ , so increasing the efficacy of contraception leads to fewer married households and more unmarried households when the marginal utility of the efficacy of contraception is greater for unmarried households than it is for married households.

Assume that there exists a points  $(p, m, z_1)$  and  $(p, m, z_2)$ ,  $z_2 > z_1$  such that  $v(p, m, z_1) = v(p, m_1, z)$  and  $v(p, m, z_2) = v(p, m_2, z)$ . These points lie on the two different indifference curves that describe unmarried and married households, but these households now differ not

income but in contraceptive efficacy. Now an increase in the efficacy of contraception will lead to fewer married households and more unmarried households—single or divorced—when

$$F' \left[ \frac{dv(p, m, z_1)}{dz} - \frac{dv(p, m, z_2)}{dz} \right] > 0.$$

This inequality shows that there will be relatively fewer married households and more unmarried households if there is diminishing marginal utility to contraceptive efficacy,  $v''(z) < 0$ . This equality will always be satisfied when a household is risk-averse, though this assumption is less restrictive than the assumption that households are risk averse.

Figure 2 shows the effect of increasing contraceptive efficacy on marriage rates. The points  $y(z_1)$  and  $y(z_2)$  show the differences in unmarried and married household utility levels for different levels of contraceptive efficacy,  $z_2 > z_1$ . As contraception becomes more effective, the utility premium of marriage is reduced, so there is less reason to get and stay married and more reason to remain or become unmarried.

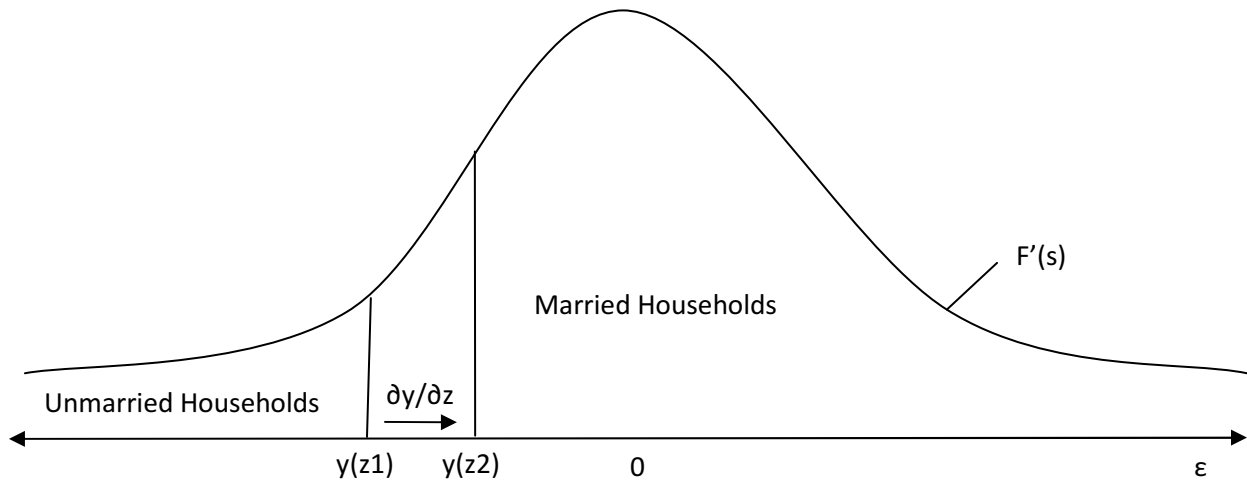


Figure 2. Effect of contraceptive efficacy on marriage rates.

For a risk averse household, let  $v(z) = u(w(z))$ , where  $w(z)$  is a linear function of  $z$  measure the value of  $z$  in terms of wealth and  $u(w)$  is the household utility function. Then

$$v''(z) = u''(w(z))[w'(z)]^2 + u'(w(z))w''(z)$$

Since  $w(z)$  is a linear function,  $w''(z) = 0$  and the second equation reduces to  $v''(z) = u''(w(z))[w'(z)]^2$ . When households are risk averse, i.e. under either constant or relative risk-aversion,  $u''(w(z)) < 0$ , which implies  $v''(z) < 0$ . The following results: if households are risk averse, then an increase in contraceptive efficacy leads to a decrease in marital stability, in the sense that the married couples are more likely to separate in any given period of time. This would correspond with various marriage trends: later ages of first marriages, lower marriage rates, and higher divorce rates, i.e. what sociologists refer to as the retreat from marriage.

The decision to marry is the decision to pool income, so cohabiting households (if they share income) are classified here as married. Of course, formally married couples are distinguished from cohabiting couples by a public promise of lifelong fidelity, but in other respects households of cohabiting couples are indistinguishable from those that are formally married. Empirically, cohabiting households are less stable than formally married households. If cohabitation can be thought of as a less stable form of marriage, then the prediction that increasing contraceptive efficacy leads to increased marital instability means that it will lead to fewer formally married households and more cohabiting ones.

### **A Comparison of the Income and Contraceptive Effects**

Let  $u(s, k, x) = u_1(s) + u_2(k) + u_3(x)$  be the utility function for the consumer, so there is no interaction in preferences between sex, children, and other goods. The contraceptive effect on children dominates the income effect if and only if the following condition holds:

$$\frac{\partial k^*}{\partial z} \Delta z + \frac{\partial k^*}{\partial m} \Delta m < 0$$

Let the cost of sex be zero. For the separable utility function, this condition becomes

$$\frac{-\beta}{|H|} p_x^2 u_{ss} s \Delta z < -\frac{\beta}{|H|} \beta (1-z)^2 p_k u_{xx} \Delta m$$

Multiplying by  $-\frac{1}{\beta} |H|$  gives

$$p_x^2 u_{ss} s \Delta z < \beta (1-z)^2 p_k u_{xx} \Delta m$$

When the fertility constraint is binding,  $(1-z)\beta = \frac{k}{s}$ , which can be substituted to show

$$p_x^2 u_{ss} s \Delta z < \frac{k}{s} (1-z) p_k u_{xx} \Delta m$$

and

$$s^2 p_x^2 u_{ss} \Delta z < k (1-z) p_k u_{xx} \Delta m$$

By assumption,  $s > k$ , so this expression will likely be satisfied for low levels of children and high levels of sex and contraceptive efficacy. Assuming that  $u_{xx}$  is negative, the effects of contraception will dominate the income effects when

$$\left[ \left( \frac{p_x s}{m} \right)^2 \frac{u_{ss}}{u_{xx}} \right] \frac{\Delta z}{(1-z)} > \frac{p_k k}{m} \frac{\Delta m}{m}$$

This equation says that the contraceptive effect will dominate the income effect when the square of the budget share of sex times the proportional reduction in contraceptive inefficacy is greater than the budget share of children times the proportional increase in income. The bracketed term on the left is the square of the budget share of sex, where the price of sex computed indirectly with the price of other goods and the ratio of the second derivatives in sex and other goods. A demonstration that this is a measure of the implicit price of sex is provided in the appendix. This bracketed term is not the actual cost of sex, but value of sex in terms of other goods; sex does not have a direct cost in the model, so its actual budget share is zero. This term can exceed one, since

it is not bound by the budget. The second left-hand side term is reduction in contraceptive inefficacy. While this cannot exceed one, it can be very large in percentage terms, e.g. an increase in contraceptive efficacy from 90 to 95% implies  $\frac{\Delta z}{1-z} = \frac{5\%}{10\%} = 50\%$ . The first right-hand side term is the budget share of children, and this is always less than one, and the second right-hand side term is the percentage increase in income, which is generally less than one for short time horizons but can exceed one over longer time periods, such as generations or centuries.

## **Discussion**

We begin by summarizing the predictions of the model and discuss them in light of historical data on the second demographic transition. We conclude with a discussion of some clear weaknesses of the model, and various extensions which could improve its value.

### *Predictions*

The three predictions of the model are that increasing contraceptive efficacy leads to increased demand for sex among all households, decreased demand for children among married households and increased demand among unmarried ones, and a decrease in the proportion of households that are married. Translating these predictions from the static context of the model to historical data implies an increase in overall sexual activity, a decrease in the fertility rate, an increase in non-marital births, delayed age of first marriage, fewer formally married households and increased marital instability—all of the trends which have been identified as part of the second demographic transition.

First, the model predicts that an increase in contraceptive efficacy will lead to an increase in sexual activity. An indirect measure of the level of sexual activity is the incidence of sexually transmitted infections (STIs). Incidence reports of incurable viral STIs show strong upward trends consistent with a significant increase in the level of sexual activity in the United States. According to the CDC, there were 28.5 reported cases of genital warts per 100,000 women in 1966 and 78.0 reported cases in 2000. Reported cases of herpes also increased from 9.7 (per 100,000 women) in 1966 to 63.4 in 2000. Data on Chlamydia, human papilloma virus (HPV), and HIV also show dramatic upward trends, though for HPV and HIV data from 1960 is not applicable. Direct measures of sexual activity (survey data) also revealed upward trends in reported sexual activity, especially among the young and unmarried.<sup>17</sup>

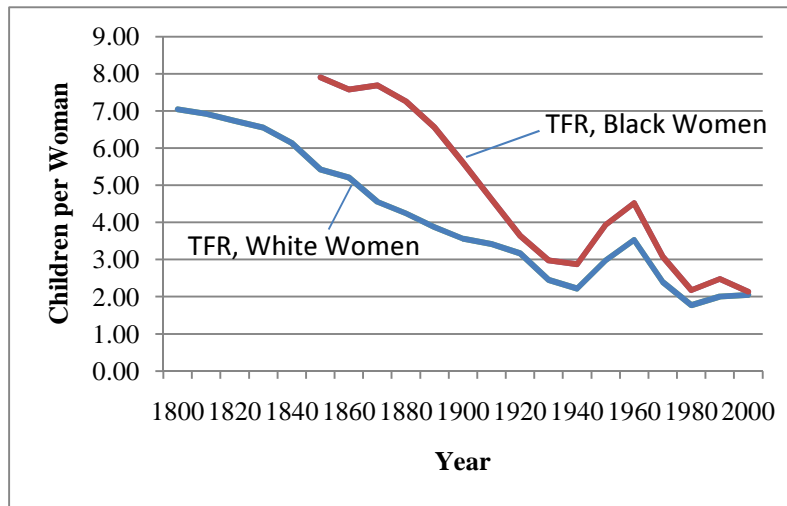


Figure 3. Total fertility rates for the US by decade, 1800-2000.<sup>18</sup>

Second, the model predicts a decrease in the overall fertility rate. Figure 3 shows that the total fertility rate (TFR) has fallen by a third, from about three to two children per woman, since

<sup>17</sup> The sharpest increases were documented in the first decade after the Pill was made available. Data from the mid-late 1980s showed a leveling off and even some modest declines in reported sexual intercourse. See Hofferth, Kahn and Baldwin (1987) for a treatment of non-marital sexual activity in the post-Pill era, and also Goldin and Katz (2000).

<sup>18</sup> Source: Haines, 2008.

the 1960s. There is a vigorous debate about whether the “normal” pre-Pill TFR would have been more like the wartime lows precipitated by the first and second world wars, or more like the post-war boom, or something different altogether. Without taking a position on the wartime patterns, we argue that the contraceptive revolution should be credited with the post-1960s downward slope in fertility. Bailey (2009) provides careful empirical support for this hypothesis utilizing state-level variation in access to the Pill.

Third, the model predicts an increase in non-marital births. As discussed in the introduction, the percent of live births to unmarried mothers increased from 10.7% in 1970 to 33.2% in 2000 and then to 36.9% in 2005. The non-marital birthrate has increased not just for the nation as a whole, but for every racial category of women. Among white women, the percent of children born to unmarried women increased from 5.5% in 1970 to 31.7% in 2005, a six-fold increase; for black women, the percent nearly doubled from 37.5% to 69.3%; and for Hispanic women, the rate more than doubled, from 23.6% in 1980 to 48% in 2005.<sup>19</sup> These trends are consistent with the predictions of the model, since all women have had access to increasingly effective contraception for the time period.

Finally, this model predicts later marriage rates, lower marriage rates, increased marital instability (more divorce and more cohabitation). These predictions are also consistent with the historical trends since 1950. Bramlett and Mosher (2002) state that

“In the United States during second half of the twentieth century, the proportion of people’s lives spent in marriage declined due to postponement of marriage to later ages and higher rates of divorce. The increase in non-marital cohabiting has also contributed to the decline in the proportion of peoples’ lives spent in marriage. Increasing rates of cohabitation have largely offset decreasing rates of marriage.”<sup>20</sup>

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<sup>19</sup> National Center for Health Statistics, 2008, Table 10, p. 172.

<sup>20</sup> Bramlett and Mosher 2002, p. 4.

This observation by Bramlett and Mosher not only confirms that the predictions of the model are consistent with historical trends, but they also situate those trends in the second half of the twentieth century, which is consistent with our hypothesis that these trends may be seen as the outcome of utility-maximizing households in the presence of a relaxed fertility constraint due to the advent of the Pill in 1960.

## **Conclusion**

The model developed here presents a description of how marginal changes in contraceptive efficacy might affect household decisions regarding marriage, sex, and children in a neo-classical framework. Increased contraceptive efficacy increases demand for sex across both married and unmarried households. This yields more children among unmarried households and fewer children among married ones, generating both an increase in non-marital births and a plausible decrease in the overall fertility rate. Finally, a significant increase in contraceptive efficacy reduces the utility differential between married and unmarried couples, perhaps explaining the so-called retreat from marriage, which is evident in later ages of first marriage, lower marriage rates, higher divorce rates, and increased rates of cohabitation. Each of these predictions of the model is consistent with the historical trends of the second demographic transition. In spite of its shortcomings (discussed below), the model presented here seems to provide a plausible basis for explaining puzzling demographic trends, especially those which have costly impact on the lives and well-being of children.

There are several limitations to the current model that provide opportunity for further research. In the model, households have children because of the income effect. If the income effect was the only reason that households had children, then (1) wealthier single households

should still have more children than poor single households, and (2) a wealthy single household will have as much demand for children as a poor married household for some combination of income levels. Neither one of these implications of the model seems to correspond to the facts, suggesting that, while the model captures something important, it is still missing something. One potential solution is to consider the interaction of marriage with children, which could perhaps be modeled by incorporating marital status into the utility function of the household directly.

Additionally, considering evidence from Edin and Kefalas regarding the obstacles that poor women face in finding suitable marriage partners, it might be interesting to introduce differential “access” to marriage by income level. This could be done by correlating the expected variance of marital income shocks with the individual income levels. For example, high earning individuals expect a fairly stable (low-variance) marital income, while low-earning individuals might expect a more variable marital income. This would have the effect of correlating non-marriage with poverty, and non-marital births to poor households.

Another limitation of the model is the treatment of contraceptive efficacy as an exogenous variable. Making this variable endogenous would be an important extension of this research. Were contraceptive efficacy endogenous in the current version of the model, a household would always choose the maximum available because doing so minimizes the indirect costs of the fertility constraint to household utility and it can be done without cost. However, many forms of contraception, while relatively inexpensive, have high indirect costs, such as diminishing the enjoyment of sex itself (e.g. condoms) or other undesirable side effects (e.g. IUDs). There may be trade-offs between contraceptive efficacy, the marginal utility of sex, and other side effects which determine which form of contraception households decide to use. This trade-off could be captured by incorporating contraceptive efficacy into the household utility function and the

budget constraint. It also suggests the existence of an efficient “contraceptive frontier,” where forms of contraception away from the frontier become obsolete over time, e.g. intrauterine devices (IUDs).

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## Appendix.

### *The Substitution Effect*

Consider the problem

$$\begin{aligned} e(p, v, z) &= \min_{s,k,x} p_s s + p_k k + p_x x \\ \text{s. t. } &v \leq u(s, k, x) \\ &(1 - z)\beta s \leq k. \end{aligned}$$

Let  $(s^H, k^H, x^H, \theta^H, \mu^H)$  be the solution to this problem. These demand functions are analogs of the Hicksian demands for the ordinary expenditure minimization problem. The Lagrangian for this problem is

$$L = p_s s + p_k k + p_x x + \theta(v - u(s, k, x)) + \mu(k - (1 - z)\beta s).$$

The system of comparative statics for this problem is

$$\begin{bmatrix} -\theta u_{ss} & -\theta u_{sk} & -\theta u_{sx} & -u_s & -(1-z)\beta \\ -\theta u_{ks} & -\theta u_{kk} & -\theta u_{kx} & -u_k & 1 \\ -\theta u_{xs} & -\theta u_{xk} & -\theta u_{xx} & -u_x & 0 \\ -u_s & -u_k & -u_x & 0 & 0 \\ -(1-z)\beta & 1 & 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} \partial s^H / \partial p_s \\ \partial k^H / \partial p_s \\ \partial x^H / \partial p_s \\ \partial \theta^H / \partial p_s \\ \partial \mu^H / \partial p_s \end{bmatrix} = \begin{bmatrix} -1 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}$$

Let  $H'$  be the left-hand side matrix. The second-order sufficiency conditions imply that  $|H'| > 0$ , since  $H'$  is a third-order bordered principal minor to the minimization problem.

The changing the price of sex leads to a change in demand for sex of

$$\frac{\partial s^H}{\partial p_s} = \frac{-u_x^2}{|H'|} < 0.$$

An increase in the price of sex leads to a decrease in the Hicksian demand for it.

The changing the price of sex leads to a change in demand for children of

$$\frac{\partial k^H}{\partial p_s} = \frac{-(1-z)\beta u_x^2}{|H'|} < 0.$$

An increase in the price of sex leads to an increase in the Hicksian demand for children.

The pure substitution effect can also be recovered from the original problem. Consider the system

$$\begin{bmatrix} u_{ss} & u_{sk} & u_{sx} & -p_s & -(1-z)\beta \\ u_{ks} & u_{kk} & u_{kx} & -p_k & 1 \\ u_{xs} & u_{xk} & u_{xx} & -p_x & 0 \\ -p_s & -p_k & -p_x & 0 & 0 \\ -(1-z)\beta & 1 & 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} \partial s^*/\partial p_s \\ \partial k^*/\partial p_s \\ \partial x^*/\partial p_s \\ \partial \lambda^*/\partial p_s \\ \partial \mu^*/\partial p_s \end{bmatrix} = \begin{bmatrix} \lambda \\ 0 \\ 0 \\ -s \\ 0 \end{bmatrix} = \begin{bmatrix} \lambda \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ 0 \\ -s \\ 0 \end{bmatrix}$$

The two vectors on the right-hand side correspond to the substitution and income effects.

An increase in the price of sex leads to a change in demand for sex of

$$\frac{\partial s^H}{\partial p_s} = \frac{\lambda p_x^2}{|H|} < 0.$$

An increase in the price of sex leads to a decrease in the Hicksian demand for it.

An increase in price of sex leads to a change in demand for children of

$$\frac{\partial k^H}{\partial p_s} = \frac{(1-z)\beta u_x^2}{|H|} < 0.$$

An increase in the price of sex leads to an increase in demand for children.

### *The Child Effect*

The pure child effect can be illustrated by considering the problem

$$\begin{aligned} & \max_{s,k,x} u(s, k, x) \\ & s. t. p_s s + p_k k + p_x x \leq m \\ & \quad k_l \leq k. \end{aligned}$$

Households choose sex, children, and goods to maximize utility subject to a budget constraint and a lower bound to the number of children. The nonnegativity constraints are implicit and assumed to be nonbinding, and a solution is assumed to for all choices of  $k_l$ , including  $k_l = 0$ .

The Lagrangian for this problem is

$$L = u(s, k, x) + \lambda(m - p_s s - p_k k - p_x x) + \gamma(k - k_l).$$

Let  $(\tilde{s}, \tilde{k}, \tilde{x}, \tilde{\lambda}, \tilde{\gamma})$  be the solution to the maximization problem. The system of equations for comparative statics analysis is

$$\begin{bmatrix} u_{ss} & u_{sk} & u_{sx} & -p_s & 0 \\ u_{ks} & u_{kk} & u_{kx} & -p_k & 1 \\ u_{xs} & u_{xk} & u_{xx} & -p_x & 0 \\ -p_s & -p_k & -p_x & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} \partial \tilde{s} / \partial k_l \\ \partial \tilde{k} / \partial k_l \\ \partial \tilde{x} / \partial k_l \\ \partial \tilde{\lambda} / \partial k_l \\ \partial \tilde{\gamma} / \partial k_l \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 1 \end{bmatrix}$$

Let  $\tilde{H}$  be the left-hand side matrix. The second-order sufficiency conditions imply that  $|\tilde{H}| < 0$ , since  $\tilde{H}$  is a third-order bordered principal minor to the maximization problem.

The effect of an increase in the lower bound on the demand for children is

$$\frac{\partial \tilde{k}}{\partial k_l} = \frac{-1}{|\tilde{H}|} \begin{vmatrix} u_{ss} & u_{sx} & -p_s \\ u_{xs} & u_{xx} & -p_x \\ -p_s & -p_x & 0 \end{vmatrix} > 0.$$

The matrix is a second-order bordered principal minor, so the second-order sufficiency conditions imply that its determinant is positive. Increasing the lower bound for the number of children, by itself, unambiguously leads to an increase in the demand for children. Conversely, decreasing the lower bound, one of the consequences of increased contraceptive efficacy, reduces demand for children.

The effect of an increase in the lower bound on the demand for sex is

$$\frac{\partial \tilde{s}}{\partial k_l} = \frac{1}{|\tilde{H}|} \begin{vmatrix} u_{sk} & u_{sx} & -p_s \\ u_{xk} & u_{xx} & -p_x \\ -p_k & -p_x & 0 \end{vmatrix}.$$

This effect is ambiguous, and depends on how children and sex are related in the utility function.

Inspection shows that this matrix is identical to the one for  $\frac{\partial s^*}{\partial z}$ .

### *The Sex Effect*

The pure sex effect can be illustrated by considering the problem

$$\begin{aligned} & \max_{s,k,x} u(s, k, x) \\ & s. t. p_s s + p_k k + p_x x \leq m \\ & \quad s \leq s_h. \end{aligned}$$

Households choose sex, children, and goods to maximize utility subject to a budget constraint and an upper bound to the amount of sex. The nonnegativity constraints are implicit and assumed to be nonbinding, and a solution is assumed to exist for all choices of  $s_h$ , including  $s_h = \infty$ . The Lagrangian for this problem is

$$L = u(s, k, x) + \lambda(m - p_s s - p_k k - p_x x) + \delta(s_h - s).$$

Let  $(\hat{s}, \hat{k}, \hat{x}, \hat{\lambda}, \hat{\delta})$  be the solution to the maximization problem. The system of equations for comparative statics analysis is

$$\begin{bmatrix} u_{ss} & u_{sk} & u_{sx} & -p_s & -1 \\ u_{ks} & u_{kk} & u_{kx} & -p_k & 0 \\ u_{xs} & u_{xk} & u_{xx} & -p_x & 0 \\ -p_s & -p_k & -p_x & 0 & 0 \\ -1 & 0 & 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} \partial \hat{s} / \partial s_h \\ \partial \hat{k} / \partial s_h \\ \partial \hat{x} / \partial s_h \\ \partial \hat{\lambda} / \partial s_h \\ \partial \hat{\delta} / \partial s_h \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ -1 \end{bmatrix}$$

Let  $\hat{H}$  be the left-hand side matrix.  $\hat{H}$  is a third-order bordered principal minor to the maximization problem, so the second-order sufficiency conditions imply that  $|\hat{H}| < 0$ .

The effect of an increase in the lower bound on the demand for children is

$$\frac{\partial \hat{s}}{\partial s_h} = \frac{-1}{|\hat{H}|} \begin{vmatrix} u_{kk} & u_{kx} & -p_k \\ u_{xk} & u_{xx} & -p_x \\ -p_k & -p_x & 0 \end{vmatrix} > 0.$$

The matrix is a second-order bordered principal minor, so the second-order sufficiency conditions imply that its determinant is positive. Increasing the upper bound for sex, by itself, leads to an increase in demand for sex. This is part of the effect of more effective contraception on demand for sex.

The effect of an increase in the lower bound on the demand for children is

$$\frac{\partial \hat{k}}{\partial s_h} = \frac{1}{|\hat{H}|} \begin{vmatrix} u_{ks} & u_{kx} & -p_k \\ u_{xs} & u_{xx} & -p_x \\ -p_s & -p_x & 0 \end{vmatrix}.$$

This effect depends on how children and sex are related in the utility function and is theoretically ambiguous. Inspection shows that this matrix is identical to the one for  $\frac{\partial k^*}{\partial z}$ .

### Separable Utility

Let  $u(s, k, x) = s^a + k^b + x^c$  be the utility function for the consumer. The corresponding system of equations for determining income effects is:

$$\begin{bmatrix} u_{ss} & 0 & 0 & -p_s & -\beta(1-z) \\ 0 & u_{kk} & 0 & -p_k & 1 \\ 0 & 0 & u_{xx} & -p_x & 0 \\ -p_s & -p_k & -p_x & 0 & 0 \\ -\beta(1-z) & 1 & 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} \partial s^*/\partial m \\ \partial k^*/\partial m \\ \partial x^*/\partial m \\ \partial \lambda^*/\partial m \\ \partial \delta^*/\partial m \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \\ -1 \\ 0 \end{bmatrix}$$

Then the income effect is found by

$$\frac{\partial s^*}{\partial m} = \frac{1}{|H|} \begin{vmatrix} 0 & 0 & 0 & -p_s & -\beta(1-z) \\ 0 & u_{kk} & 0 & -p_k & 1 \\ 0 & 0 & u_{xx} & -p_x & 0 \\ -1 & -p_k & -p_x & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \end{vmatrix}$$

Simplifying using Cramer's rule gives

$$\frac{\partial s^*}{\partial m} = \frac{1}{|H|} [\beta(1-z)p_k + p_s]u_{xx}$$

Similarly, the income effect on kids is found by

$$\frac{\partial k^*}{\partial m} = \frac{1}{|H|} \begin{vmatrix} u_{ss} & 0 & 0 & -p_s & -\beta(1-z) \\ 0 & 0 & 0 & -p_k & 1 \\ 0 & 0 & u_{xx} & -p_x & 0 \\ -p_s & -1 & -p_x & 0 & 0 \\ -\beta(1-z) & 0 & 0 & 0 & 0 \end{vmatrix}$$

Cramer's rule gives

$$\frac{\partial k^*}{\partial m} = \frac{1}{|H|} [\beta(1-z)p_k + p_s] \beta(1-z) u_{xx}$$

The corresponding system of equations for determining the effects of contraceptive efficacy,  $z$ , is as follows:

$$\begin{bmatrix} u_{ss} & 0 & 0 & -p_s & -\beta(1-z) \\ 0 & u_{kk} & 0 & -p_k & 1 \\ 0 & 0 & u_{xx} & -p_x & 0 \\ -p_s & -p_k & -p_x & 0 & 0 \\ -\beta(1-z) & 1 & 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} \partial s^*/\partial z \\ \partial k^*/\partial z \\ \partial x^*/\partial z \\ \partial \lambda^*/\partial z \\ \partial \delta^*/\partial z \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ -\beta s \end{bmatrix}$$

So the effect of contraceptive efficacy on sex will be

$$\frac{\partial s^*}{\partial z} = \frac{1}{|H|} \begin{vmatrix} 0 & 0 & 0 & -p_s & -\beta(1-z) \\ 0 & u_{kk} & 0 & -p_k & 1 \\ 0 & 0 & u_{xx} & -p_x & 0 \\ 0 & -p_k & -p_x & 0 & 0 \\ -\beta s & 1 & 0 & 0 & 0 \end{vmatrix}$$

Cramer's rule gives

$$\frac{\partial s^*}{\partial z} = \frac{1}{|H|} [\beta(1-z)p_x^2 u_{kk} + [p_s p_k - \beta(1-z)p_k^2] u_{xx}] \beta s$$

The effect of contraceptive efficacy on kids will be

$$\frac{\partial k^*}{\partial z} = \frac{1}{|H|} \begin{vmatrix} u_{ss} & 0 & 0 & -p_s & -\beta(1-z) \\ 0 & 0 & 0 & -p_k & 1 \\ 0 & 0 & u_{xx} & -p_x & 0 \\ -p_s & 0 & -p_x & 0 & 0 \\ -\beta(1-z) & -\beta s & 0 & 0 & 0 \end{vmatrix}$$

Cramer's rule gives

$$\frac{\partial k^*}{\partial z} = \frac{-1}{|H|} [p_x^2 u_{ss} - [p_s^2 + \beta(1-z)p_s p_k] u_{xx}] \beta s$$

### *Implicit Prices*

Assume that utility is linearly separable and consider the equation

$$\frac{u_x}{p_x} = \lambda$$

This equation is implied by the first-order conditions to the ordinary consumer's problem. It must be satisfied for all goods and for all income levels, so

$$\frac{u_x(x(m))}{p_x} \equiv \lambda(m)$$

Taking derivatives of both sides with respect to  $m$  shows

$$\frac{1}{p_x} u_{xx} \frac{\partial x}{\partial m} \equiv \frac{\partial \lambda}{\partial m}$$

But, from the budget constraint,

$$m = p_x x + p_y y + p_z z$$

so

$$\frac{\partial m}{\partial x} = p_x$$

and

$$\frac{\partial x}{\partial m} = \frac{1}{p_x}$$

Therefore

$$\frac{u_{xx}}{p_x^2} \equiv \frac{\partial \lambda}{\partial m}$$

This is true for all goods, so

$$\frac{u_{xx}}{p_x^2} = \frac{u_{yy}}{p_y^2} = \frac{u_{zz}}{p_z^2} = \frac{\partial \lambda}{\partial m}$$

From this the square of one price can be computed in terms of the square of another as

$$p_x^2 = \frac{u_{xx}}{u_{yy}} p_y^2.$$

## Notes

### *Effects of Contraceptive Improvements on Marriage*

Contraception affects the model through the fertility constraint

$$(1 - z)\beta s(z) = k(z).$$

For a new contraceptive level  $z' = tz$ ,

$$(1 - tz)\beta s(tz) = k(tz).$$

Adding and subtracting  $z$  in the parentheses shows

$$(1 - tz + z - z)\beta s(tz) = k(tz).$$

This can be rearranged to show

$$(1 - z)\beta s(tz) - k(tz) = z(t - 1)\beta s(tz).$$

Let  $s(tz) = s(z) + ds$  and  $k(tz) = k(z) + dk$ .

$$(1 - z)\beta ds = z(t - 1)\beta s(tz) + dk.$$

This equation shows that the change in sex proportional to the change in children plus a positive term. There are only three possibilities: demand increases for sex and decreases for children, demand increases for both, or demand decreases for both. It is not possible that demand for sex can decrease and children increase. Furthermore, it is implausible that demand for both sex and children would decrease, since the substitution effect would ordinarily lead to an increase in demand for sex.

For some  $a$  and  $c$ , let  $s(tz) = t^a s(z)$  and  $k(tz) = t^b k(z)$ . This equation becomes

$$(1 - z)\beta t^a s(z) - t^b k(z) = z(t - 1)\beta s(tz).$$

Substituting in the original equation shows

$$(t^a - t^b)(1 - z)\beta s(z) + t^b [(1 - z)\beta s(z) - k(z)] = z(t - 1)\beta s(tz).$$

The bracketed term is zero, so this equation becomes

$$(t^a - t^b)(1 - z)\beta s(z) = z(t - 1)\beta s(tz).$$

For  $t > 1$ , the right hand side is positive, so  $t^a > t^b$  and  $a > b$ ; for  $0 < t < 1$ , the right hand side is negative, so  $t^a < t^b$  and  $a > b$ . Assuming that  $s(tz)$  is a continuous function of  $t$  in a small neighborhood around  $tz$ , the derivative of both sides of  $s(tz) = t^a s(z)$  with respect to  $t$  shows  $\frac{\partial s}{\partial z} z = at^{a-1} s(z)$ . Dividing both sides of this expression by  $s(z)$  and evaluating it where  $t = 1$  shows  $a$  is the elasticity of sex with respect to contraceptive efficacy,  $\varepsilon_{s,z}$ . Similarly,  $b$  is the elasticity of children with respect to contraceptive efficacy,  $\varepsilon_{k,z}$ . The result is then that, with respect to changes in contraceptive efficacy, demand for sex is more elastic than demand for children,  $\varepsilon_{s,z} > \varepsilon_{k,z}$ .

The budget constraint can be used to evaluate the elasticity of demand for other goods with respect to contraceptive efficacy. Taking derivatives of the budget with respect to  $z$  shows

$$p_s \frac{\partial s}{\partial z} + p_k \frac{\partial k}{\partial z} + p_x \frac{\partial x}{\partial z} = 0$$

Let  $\sigma_s$ ,  $\sigma_k$ , and  $\sigma_x$  be the budget shares of sex, children, and other goods. This equation can be rearranged to show

$$\sigma_s \varepsilon_{s,z} + \sigma_k \varepsilon_{k,z} + \sigma_x \varepsilon_{x,z} = 0$$

When the price of sex is negligible,  $\sigma_s \approx 0$ , and this equation can be rearranged to show

$$\varepsilon_{x,z} = -\frac{\sigma_k}{\sigma_x} \varepsilon_{k,z}$$

The elasticity of other goods with respect to contraceptive efficacy is proportional to the elasticity of children and the ratio of their budget shares, and they have opposite sign. There were two realistic possibilities: demand for children can increase or decrease, but demand for sex is expected to increase in either case. In the former case, households have more sex and more children, and consumption of other goods decreases; in the latter, households have more sex and fewer children, and consumption of other goods increases. We argue that the first describes the

effect of contraceptive improvements on unmarried households, and the second its effect on married households.

### *CES Utility*

Let  $u(s, k, x) = s^a + k^b + x^c$  be the utility function for the consumer.

The income effect for kids is

$$\frac{\partial k^*}{\partial m} = \frac{-1}{|H|} [\beta(1-z)p_k + p_s] \beta(1-z)b(1-b)x^{c-2} > 0$$

The income effect for sex is

$$\frac{\partial s^*}{\partial m} = \frac{-1}{|H|} [\beta(1-z)p_k + p_s] b(1-b)x^{b-2} > 0$$

The effect of increasing contraceptive efficacy on kids is

$$\frac{\partial k^*}{\partial z} = \frac{1}{|H|} [p_x^2 a(1-a)s^{a-2} - [p_s^2 + \beta(1-z)p_s p_k] (c(1-c)x^{c-2})] \beta s$$

The effect of increasing contraceptive efficacy on sex is

$$\frac{\partial s^*}{\partial z} = \frac{-1}{|H|} [\beta(1-z)p_x^2 b(1-b)k^{b-2} + [p_s p_k - \beta(1-z)p_k^2] c(1-c)x^{c-2}] \beta s$$

### *Income Effects*

In the ordinary consumer problem without the fertility constraint, the system of equations for comparative statics analysis is

$$\begin{bmatrix} u_{ss} & u_{sk} & u_{sx} & -p_s \\ u_{ks} & u_{kk} & u_{kx} & -p_k \\ u_{xs} & u_{xk} & u_{xx} & -p_x \\ -p_s & -p_k & -p_x & 0 \end{bmatrix} \begin{bmatrix} \partial s^* / \partial m \\ \partial k^* / \partial m \\ \partial x^* / \partial m \\ \partial \lambda^* / \partial m \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \\ -1 \end{bmatrix}.$$

Let H be the left-hand side matrix, where the second-order sufficiency conditions imply  $|H| < 0$ .

$$\begin{aligned}\frac{\partial s^*}{\partial m} &= \frac{1}{|H|} \left[ -p_s \begin{vmatrix} u_{kk} & u_{kx} \\ u_{xk} & u_{xx} \end{vmatrix} + p_k \begin{vmatrix} u_{sk} & u_{sx} \\ u_{xk} & u_{xx} \end{vmatrix} - p_x \begin{vmatrix} u_{sk} & u_{sx} \\ u_{kk} & u_{kx} \end{vmatrix} \right] \\ \frac{\partial k^*}{\partial m} &= \frac{-1}{|H|} \left[ -p_s \begin{vmatrix} u_{ks} & u_{kx} \\ u_{xs} & u_{xx} \end{vmatrix} + p_k \begin{vmatrix} u_{ss} & u_{sx} \\ u_{xs} & u_{xx} \end{vmatrix} - p_x \begin{vmatrix} u_{ss} & u_{sx} \\ u_{ks} & u_{kx} \end{vmatrix} \right] \\ \frac{\partial x^*}{\partial m} &= \frac{1}{|H|} \left[ -p_s \begin{vmatrix} u_{ks} & u_{kk} \\ u_{xs} & u_{xk} \end{vmatrix} + p_k \begin{vmatrix} u_{ss} & u_{sk} \\ u_{xs} & u_{xk} \end{vmatrix} - p_x \begin{vmatrix} u_{ss} & u_{sk} \\ u_{ks} & u_{kk} \end{vmatrix} \right].\end{aligned}$$

The assumptions that each of these three goods are normal implies, respectively, that

$$\begin{aligned}p_s \begin{vmatrix} u_{kk} & u_{kx} \\ u_{xk} & u_{xx} \end{vmatrix} &> p_k \begin{vmatrix} u_{sk} & u_{sx} \\ u_{xk} & u_{xx} \end{vmatrix} - p_x \begin{vmatrix} u_{sk} & u_{sx} \\ u_{kk} & u_{kx} \end{vmatrix} \\ p_k \begin{vmatrix} u_{ss} & u_{sx} \\ u_{xs} & u_{xx} \end{vmatrix} &> p_s \begin{vmatrix} u_{ks} & u_{kx} \\ u_{xs} & u_{xx} \end{vmatrix} + p_x \begin{vmatrix} u_{ss} & u_{sx} \\ u_{ks} & u_{kx} \end{vmatrix} \\ p_x \begin{vmatrix} u_{ss} & u_{sk} \\ u_{ks} & u_{kk} \end{vmatrix} &> p_k \begin{vmatrix} u_{ss} & u_{sk} \\ u_{xs} & u_{xk} \end{vmatrix} - p_s \begin{vmatrix} u_{ks} & u_{kk} \\ u_{xs} & u_{xk} \end{vmatrix}.\end{aligned}$$

The sum of the income elasticities weighted by budget share must equal one. Let  $w_s$ ,  $w_k$ , and  $w_x$  budget shares and  $e_{s,m}$ ,  $e_{k,m}$ , and  $e_{x,m}$  the income elasticities of sex, children, and other goods, so

$$w_s e_{s,m} + w_k e_{k,m} + w_x e_{x,m} = 1.$$

Rearranging this equation shows

$$e_{k,m} = \frac{1}{w_k} - \frac{w_s e_{s,m}}{w_k} - \frac{w_x e_{x,m}}{w_k}.$$

Because other goods are assumed to be normal, the right-hand side term is positive, so a simple upper bound for the income elasticity of children is

$$e_{k,m} \leq \frac{1}{w_k}.$$

## Primal-Dual Analysis

The primal-dual problem is

$$\begin{aligned} 0 &= \max_z u(s, k, x) - v(p, m, z) \\ \text{s. t. } &p_s s + p_k k + p_x x \leq m \\ &(1 - z)\beta s \leq k. \end{aligned}$$

The Lagrangian for this problem is

$$L = u(s, k, x) - v(p, m, z) + \lambda(m - p_s s - p_k k - p_x x) + \mu(k - (1 - z)\beta s).$$

The first-order condition is

$$-v_z + \mu\beta s = 0.$$

From the Envelope Theorem,

$$v_z(z) = \mu(z)\beta s(z).$$

## Duality

Let the optimal choice of sex for the three problems be  $s^*(p, m, z)$ ,  $\tilde{s}(p, m, k_l)$ , and  $\hat{s}(p, m, s_h)$ . Then

$$s^*(p, m, z) \equiv \tilde{s}(p, m, (1 - z)\beta s^*(p, m, z))$$

$$s^*(p, m, z) \equiv \hat{s}(p, m, s^*(p, m, z))$$

Taking derivatives shows

$$\frac{\partial s^*}{\partial z} \equiv \frac{\partial \tilde{s}}{\partial k_l} \left[ -\beta s^* + (1 - z)\beta \frac{\partial s^*}{\partial z} \right] \equiv \frac{\partial \tilde{s}}{\partial k_l} \frac{\partial k^*}{\partial z}$$

$$\frac{\partial s^*}{\partial z} \equiv \frac{\partial \hat{s}}{\partial s_h} \frac{\partial s^*}{\partial z}$$

The second identity implies that

$$\frac{\partial \hat{s}}{\partial s_h} \equiv 1$$

Let the optimal choice of children for the three problems be  $k^*(p, m, z)$ ,  $\tilde{k}(p, m, k_l)$ , and  $\hat{k}(p, m, s_h)$ . Then

$$k^*(p, m, z) \equiv \tilde{k}(p, m, k^*(p, m, z))$$

$$k^*(p, m, z) \equiv \hat{k}(p, m, s^*(p, m, z)).$$

Taking derivatives shows

$$\frac{\partial k^*}{\partial z} \equiv \frac{\partial \tilde{k}}{\partial k_l} \frac{\partial k^*}{\partial z}$$

$$\frac{\partial k^*}{\partial z} \equiv \frac{\partial \hat{k}}{\partial s_h} \frac{\partial s^*}{\partial z}.$$

The first identity implies

$$\frac{\partial \tilde{k}}{\partial k_l} \equiv 1.$$

## The General Problem

Consider the problem

$$\begin{aligned} v(p, m, z) &= \max_{s, k, x} u(s, k, x) \\ \text{s. t. } &p_s s + p_k k + p_x^T x \leq m \\ &(1 - z)\beta s \leq k. \end{aligned}$$

Here  $x$  is a vector of goods. Solutions to this problem have the form  $s^*(p, m, z)$ ,  $k^*(p, m, z)$ , and  $x^*(p, m, z)$ .

Now consider the constrained problem

$$\begin{aligned} v(p, m, z) &= \max_{s, k, x} u(s, k, x) \\ \text{s. t. } &p_s s + p_k k + p_x^T x \leq m \\ &s \leq s_h \\ &k_l \leq k \end{aligned}$$

Solutions to this problem have the form  $s^c(p, m, s_h, k_l)$ ,  $k^c(p, m, s_h, k_l)$ , and  $x^c(p, m, s_h, k_l)$ .